Technical report #10

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Introduction

Critical probability in traffic modelling and control

abstract

Percolation has been playing a big role in many fields of Physics, Chemistry as well as Forestry and Biology. But upto this time its application to traffic modelling and control has not been mentioned in existing literature, contrary to its inherently similar in nature to the field. This work is an attempt to introduce it to this application.

Introduction

Percolation says that for any kind of networks of connections among components, namely vertices and bonds, there is one parameter with its own unique value which could be different from other or other kind of networks. This parameter is named *the percolation probability* or *the critical probability.* This critical probability or p_c marks the transition from one of the two phases into another.

Theory

In order to be succinct, only one theorem will be briefly mentioned.

Theorem 1 *For any traffic networks the state of traffic at any moment can be described as being in one of the three states namely,* free-flowing, congested *or* stand-still. *Furthermore, the free-flowing case corresponds to the case where* $p < \min(1 - p_c)$*, the congested case to the case where* $\min((1 - p_c), p_c) \leq p \leq \max(1 - p_c), (p_c)$ *and the stand-still case to* $p \geq \max(1 - p_c), (p_c)$*.*

Proof. From percolation theory we know that there exists $0 < p_c < 1$ for any networks. Obviously p_c can only have have a value within one of these three intervals, that is < 0.5, $= 0.5$ or > 0.5. Furthermore we may consider the percolation as being percolation of flowing roads as well as the percolation of jammed roads. Our interval from 0 to 1 can then be divided into three sections namely, $0 < p < \min(1 - p_c)$, $\min((1 - p_c), p_c) \leq p \leq \max(1 - p_c), (p_c)$ and $\max(1 - p_c), (p_c) < p < 0$. The first one of the three intervals above corresponds to the situation where cars have not yet percolate while space has. And thus it corresponds to the free-flowing traffic. The next one corresponds either to the situation where both cars and space has percolated or to the situation where neither has percolated depending on whether $p_c < 0.5$ or $p_c > 0.5$ originally. And thus it corresponds to the congested traffic. And the last one corresponds to the situation where cars have percolated while space has not and therefore to the stand-still traffic. The case where $p_c = 0.5$ is a special case where the second one of the three intervals considered vanishes.$\odot$

Traffic Modelling

For modelling purpose the percolation probability of the networks being considered is obtained by using the procedure shown in Figure 1.

The structures of the data mentioned in Figure 1 are given below.

Data Structure 1 *Junction Data in file named Junction. 3 datasets per record.*

J *the number assigned to this junction, of type* $\mathbb{I}^+$.

x *x-coordinate of this junction, of type* $\mathbb{R}^+$.

y *y-coordinate of this junction, of type* $\mathbb{R}^+$.

Data Structure 2 *Road Data in file named Road. 3 datasets per record.*

R *the number assigned to this road, of type* $\mathbb{I}^+$.

i *the number of the junction at one end of this road, of type* $\mathbb{I}^+$.

j *the number of the junction at the other end of this road, of type* $\mathbb{I}^+$.

Data Structure 3 *Cluster*

C *the number assigned to this cluster, of type* $\mathbb{I}^+$.

n *the amount of roads belonging to the cluster, of type* $\mathbb{I}^+$.

f *the fractal dimension of the cluster,* $\mathbb{R}^+$.

M *set of member roads of this cluster,* $M = \{m_i : m_i \in \text{(cluster } C) m_i \in \mathbb{R}, i = 1, \ldots, n\}$.

For illustrative purpose a network of an imaginary city called *City X* is considered in Figure 2 and Figure 3. Figure 3 is that of the networks in Figure 2 after the construction of a ring road. The percolation probability of the networks shown in Figure 2 was found to be approximately 0.75, averaged over 20 simulations.

The percolation probability of the networks shown in Figure 3 was found to be approximately 0.72, averaged over 20 simulations. That is the addition of the ring road reduced the p_c.

The results shows that the ring road added resulted in the reduction of the p_c. And since the p_c of this network is > 0.5 this also resulted in the reduction of the probability that this network will be in a congested situation.

Traffic Control

Let S be the situation where the percolation of roads where traffics are flowing occurs. S is namely *space percolation*. And let H be the situation where the percolation of congested roads occurs. H is namely *car percolation*. The decisional cases in Figure 4 are cases where $(S \wedge \neg H)$, $(S \wedge H) \vee (\neg S \wedge \neg H)$ and $(\neg S \wedge H)$ for cases number 1, 2 and 3 respectively. The parameter t_k in Figure 4 is the sampling time of the data. t_k could be approximately 5 minutes.

Conclusion

I hope that this work has showed the possibility of applying the percolation theory to modelling and control problems of traffic networks. More work is still needed to be done in order to deliberate upon this idea. It is important to note also that this approach, if turned out to be feasible, could be similarly applied to fields where other kinds of traffic are concerned too, to mention but one of such fields is the traffic inside the Internet.

Modelling of Traffic Congestion

abstract

Bad traffic conditions could have adverse effects on a country's economics and well-beings. News of such devastating situations as a traffical stand-still means reduced competitive edges for a developping country, discouraged tourists and an enormous amount of wasted resources. Percolation theory has long been associated with similar problems occuring in other areas of science, for example sieve-blinding and filtration. These problems essentially deals with the flow of fluids through a media which has random structure and, when the media is subjected to blockages. Similar to how a filter media can be classified according to the way it behaves at the onset of percolation, any traffical networks could be distinguished from the others by its ability to withstand heavy traffic conditions before a stand-still occurs. This work is the beginning of an attempt to model traffical networks with the idea of percolation in mind. It has been proposed here that a traffical networks can either be excellent at tolerating a stand-still or, it can boast that it rarely finds itself congested, but never both at the same time. It has been proposed that a trade-off has to be made if one were to design a new traffical networks.

Introduction

There are two kinds of lattices or tessellations. One is regular and the other random. Examples of regular tessellations are honeycomb and Kagomé lattices in 2-dimension and hexagonal tessellations in 3-dimension. Tessellations which are not regular are random [CN88, MC89, HS95]. There are random tessellations with underlying rules as a Voronoï tessellations [Boo87, Muc96, RHOG88]. Examples of these rule-specific random tesselations are patterns of cells in living tissue and structure of the universe [vdW94]. An example of these purely-random tessellations is a traffical networks.

Critical probability [JG96, Cha96, JG95] is a property which has long been associated with the study of tessellations' properties. This value is characteristic for each type of tessellations concerned and is closely related to the coordination number of the tessellations or, in the random tessellations case, the average coordination number. For all kind of tessellations finding the critical probability can be done by doing simulations. Theoretical computation [SK71] of exact values of crical probabilities is also possible but, only for certain types of regular tessellations. So for traffic networks the only two possible ways of finding the critical probability for a network of a certain city are by doing simulations on computer and, by approximating the value from the average coordination number of the network.

Topics related to traffic congestions and the states of congestion have been discussed in existing literatures [SAH97, Sis97, HW97]. In this work it has been proposed for a traffical networks of a specific city that, once the number of roads experiencing a stoppage reaches a certain value the whole city will come to a stand-still. It has also been further proposed that, in the designing of a traffical networks a trade-off has to be made between robustness to congestions and robustness to stand-stills. This means that we can have a traffic networks which is most of the time congested but never comes to an overall stand-still or, we can have a traffic networks which is most of the time flowing freely until the amount of cars increases causing it to come to an overall stand-still, but we can not have a kind of traffic networks which is free-flowing all the time and never experience a stand-still.

Study of traffic networks based on queueing theory also categorizes traffic into several states [Leu88] but, the definitions of these states are quite different from those here.

Theory

This section is organized as a number of definitions leading to main theorems. These definitions are good because they serve as definitive description to the problem as well as aid for writing a program for doing simulation.

Definition 1 *A* network *is the linkages of* vertices *by* bonds *and has* bond percolation probability *and* vertex percolation probabability *as its two invariances. A network will be denoted by* $\mathrm{Nwk}(\mathrm{Vtx}, \mathrm{Bnd}, P_{\mathrm{Vtx}}, P_{\mathrm{Bnd}})$ *where* $\mathrm{Vtx}, \mathrm{Bnd} \subset \mathbb{N} + \{+\infty\}$, $P_{\mathrm{Vtx}}, P_{\mathrm{Bnd}} \in \mathbb{R}$, $0 < P_{\mathrm{Vtx}} < 1$ *and* $0 < P_{\mathrm{Bnd}} < 1$.

Definition 2 $\mathrm{Nwk}(\mathrm{Bnd})$ *represents a network which is considered as connections of bonds at vertices while* $\mathrm{Nwk}(\mathrm{Vtx})$ *represents a network which is considered as connections of vertices by bonds.*

Definition 3 $\mathrm{Bnd}(V)$ *represents a set of all the bonds which are connected to* $v\ \forall v \in V\ V \subset \mathrm{Vtx}$. *Similarly* $\mathrm{Vtx}(B)$ *represents a set of all the vertices which are connected to* $b\ \forall b \in B\ B \subset \mathrm{Bnd}$.

Definition 4 *A bond can have either of the two states which are called* free *(*Bnd^0*) or* blocked *(*Bnd^1*). Likewise a vertex can be either* free *(*Vtx^0*) or* occupied *(*Vtx^1*).*

Definition 5 *A* cluster *is a set of either of the two components of a network, namely vertices and bonds, with each one of its members connecting to some other members. There are clusters of bonds represented by*

$$\{\mathrm{Cl}(\mathrm{Bnd}) : \forall b_1 \in \mathrm{Cl}(\mathrm{Bnd}) \exists b_2 \in \mathrm{Cl}(\mathrm{Bnd})\ s.t.\ \mathrm{Vtx}(b_1) = \mathrm{Vtx}(b2) \quad \mathrm{Cl}(\mathrm{Bnd}) \subset \mathrm{Bnd}\} \tag{1}$$

and there are clusters of vertices represented by

$$\{\mathrm{Cl}(\mathrm{Vtx}) : \forall v_1 \in \mathrm{Cl}(\mathrm{Vtx}) \exists v_2 \in \mathrm{Cl}(\mathrm{Vtx})\ s.t.\ \mathrm{Bnd}(v_1) = \mathrm{Bnd}(v2) \quad \mathrm{Cl}(\mathrm{Vtx}) \subset \mathrm{Vtx}\}. \tag{2}$$

Definition 6 *The* set of all the clusters *(of either bonds or vertices) is*

$$\{\mathrm{Tcl}(\cdot) : \mathrm{Cl}(\cdot) \subset \mathrm{Nwk}(\cdot) \Longleftrightarrow \mathrm{Cl}(\cdot) \in \mathrm{Tcl}(\cdot)\}, \tag{3}$$

where $\cdot$ *is either* Bnd *or* Vtx.

Definition 7 *Each cluster has a definite* size *which is the number of either bonds or vertices comprising it, depending on whether the cluster is a cluster of bonds or a cluster of vertices. This size is represented by*

$$\mathrm{Size}(\mathrm{Cl}) \quad \mathrm{Cl} \in \mathbb{N} + \{+\infty\} \tag{4}$$

Definition 8 *The* width *of a cluster is the dimension of the cluster in either* x*- or* y*-axis and, is represented by*

$$\mathrm{Width}(\mathrm{Cl}) \in \mathbb{R} \quad \mathrm{Cl} \in \mathbb{N} + \{+\infty\} \tag{5}$$

Definition 9 *The* largest cluster *(of either bonds or vertices) of any network is*

$$\{\,\mathrm{Mcl}(\cdot):\ \mathrm{Size}(\,\mathrm{Mcl}) = \sup(\,\mathrm{Size}(\,\mathrm{Cl}))\ \forall\,\mathrm{Cl} \in\ \mathrm{Tcl}\}\,. \tag{6}$$

Definition 10 *The* percolating cluster *of any network is*

$$\{\,\mathrm{Pclust}(\cdot):\ \mathrm{Pclust}(\cdot) =\ \mathrm{Mcl}\ \ \mathrm{Mcl}(\cdot) \longrightarrow \infty \ \textit{as}\ \mathrm{Size}(\,\mathrm{Nwk}(\cdot)) \longrightarrow \infty\}\,. \tag{7}$$

While the smallest percolating cluster *possible for any network is*

$$\mathrm{Inf}(\cdot) = \inf(\,\mathrm{Pclust}) \tag{8}$$

Definition 11 *The* percolating cluster *of a traffical network is*

$$\{\,\mathrm{Pcl}(\,\mathrm{Bnd}):\ \mathrm{Pcl}(\,\mathrm{Bnd}) =\ \mathrm{Cl}(\,\mathrm{Bnd})\quad \mathrm{Width}(\,\mathrm{Cl}(\,\mathrm{Bnd})) =\ \mathrm{Width}(\,\mathrm{Nwk}(\,\mathrm{Bnd}))\} \tag{9}$$

Definition 12 *For any network a* critical probability *is*

$$\left\{\mathrm{Pc}(\cdot):\ \mathrm{Pc}(\cdot) = \frac{\sum_{\forall\,\mathrm{Cl}\in\,\mathrm{Tcl}} \mathrm{Size}(\,\mathrm{Cl}(\cdot))}{\mathrm{Size}(\,\mathrm{Nwk}(\cdot))}\quad \mathrm{Mcl}(\cdot) =\ \mathrm{Inf}(\cdot)\right\}. \tag{10}$$

While $\mathrm{Pc}(\,\mathrm{Bnd})$ *is its* bond critical probablity *and* $\mathrm{Pc}(\,\mathrm{Vtx})$ *is its* vertex critical probability.

Definition 13 *For a traffical network a* critical probability *is*

$$\left\{\mathrm{Pc}(\,\mathrm{Bnd}):\ \mathrm{Pc}(\cdot) = \frac{\sum_{\forall\,\mathrm{Cl}\in\,\mathrm{Tcl}} \mathrm{Size}(\,\mathrm{Cl}(\,\mathrm{Bnd}))}{\mathrm{Size}(\,\mathrm{Nwk}(\,\mathrm{Bnd}))}\quad \mathrm{Mcl}(\,\mathrm{Bnd}) =\ \mathrm{Pcl}(\,\mathrm{Bnd})\right\}. \tag{11}$$

Definition 14 *A* free-flowing network *is*

$$\left\{\mathrm{Nwk}: \exists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^0))\quad \nexists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^1))\right\}. \tag{12}$$

A stand-still *is*

$$\left\{\mathrm{Nwk}: \nexists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^0))\ \ \exists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^1))\right\}. \tag{13}$$

A congested network *is*

$$\begin{gathered}\left\{\mathrm{Nwk}: \left\{\nexists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^0))\quad \nexists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^1))\right\} \bigoplus \right.\\ \left.\left\{\exists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^0))\ \ \exists\,\mathrm{Pcl}(\,\mathrm{Nwk}(\,\mathrm{Bnd}^1))\right\}\right\}.\end{gathered} \tag{14}$$

The following definitions concern with features found in typical traffical networks.

Definition 15 *A* road *is a single bond. It can be either a one-way or a two-way street but, without any branching along its length. The only two opennings are located at both ends.*

Definition 16 *At any instance a road is said to be* congested *when no cars could enter it.*

Definition 17 *A round-about is considered simply as being a vertex.*

Definition 18 *Let* $p = \frac{\sum_{\forall \mathrm{Cl} \in \mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\cdot))}{\mathrm{Size}(\mathrm{Nwk}(\cdot))}$ *then,*

$$\mathrm{Frf}(\mathrm{Nwk}(\mathrm{Bnd})) = \{\mathrm{Nwk}(\mathrm{Bnd}) : p \in [0, \min(\mathrm{Pc}, 1 - \mathrm{Pc}))\}, \tag{15}$$

$$\mathrm{Cgd}(\mathrm{Nwk}(\mathrm{Bnd})) = \{\mathrm{Nwk}(\mathrm{Bnd}) : p \in [\min(\mathrm{Pc}, 1 - \mathrm{Pc}), \max(\mathrm{Pc}, 1 - \mathrm{Pc})]\}, \tag{16}$$

and

$$\mathrm{Stp}(\mathrm{Nwk}(\mathrm{Bnd})) = \{\mathrm{Nwk}(\mathrm{Bnd}) : p \in (\max(\mathrm{Pc}, 1 - \mathrm{Pc}), 1]\}. \tag{17}$$

Theorem 2 *For a traffical network,*

$$\mathrm{Pc}(\mathrm{Bnd}^0) = \mathrm{Pc}(\mathrm{Bnd}^1), \tag{18}$$

where Bnd^0 *is a free bond or a non-congested road and,* Bnd^1 *is an occupied bond or a congested road.*

Proof
The networks remains the same whether one consider it as a network of free roads or, that of congested roads. In other words,

$$\mathrm{Nwk}(\mathrm{Bnd}^0) = \mathrm{Nwk}(\mathrm{Bnd}^1) = \mathrm{Nwk}(\mathrm{Bnd}) \tag{19}$$

When $\mathrm{Mcl}(\mathrm{Bnd}^0) = \mathrm{Pcl}(\mathrm{Bnd}^0)$,

$$\mathrm{Pc}(\mathrm{Bnd}^0) = \frac{\sum_{\forall \mathrm{Cl}(\mathrm{Bnd}^0) \in \mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^0))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}^1))}. \tag{20}$$

While when $\mathrm{Mcl}(\mathrm{Bnd}^1) = \mathrm{Pcl}(\mathrm{Bnd}^1)$,

$$\mathrm{Pc}(\mathrm{Bnd}^1) = \frac{\sum_{\forall \mathrm{Cl}(\mathrm{Bnd}^1) \in \mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^1))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}^0))}. \tag{21}$$

Now suppose instead of starting from $\mathrm{Nwk}(\mathrm{Bnd}^0)$ and keep adding Bnd^1 randokmly until $\mathrm{Mcl}(\mathrm{Bnd}^1) = \mathrm{Pcl}(\mathrm{Bnd}^1)$, we could have said from the start that we wer looking at $\mathrm{Nwk}(\mathrm{Bnd}^1)$ and added Bnd^0 randomly until $\mathrm{Mcl}(\mathrm{Bnd}^0) = \mathrm{Pcl}(\mathrm{Bnd}^0)$. From this reasoning

$$\mathrm{Mcl}(\mathrm{Bnd}^1) = \mathrm{Pcl}(\mathrm{Bnd}^1) = \mathrm{Pcl}(\mathrm{Bnd}^0) = \mathrm{Mcl}(\mathrm{Bnd}^0). \tag{22}$$

From Equation 19 we have

$$\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}^0)) = \mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}^1)) = \mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd})). \tag{23}$$

Thus it follows from Equation 20 and Equation 21 that

$$\mathrm{Pc}(\mathrm{Bnd}^0) = \mathrm{Pc}(\mathrm{Bnd}^1). \tag{24}$$

QED.

Theorem 3 *For a traffical network, when* $\mathrm{Mcl}(\mathrm{Bnd}^1) = \mathrm{Pcl}(\mathrm{Bnd}^1)$

$$\mathrm{Pc}(\mathrm{Bnd}^0) = P_{\mathrm{Bnd}^0} = 1 - \frac{\sum_{\forall\,\mathrm{Cl}\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^0))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}))}. \tag{25}$$

Proof
When $\mathrm{Mcl}(\mathrm{Bnd}^1) = \mathrm{Pcl}(\mathrm{Bnd}^1)$ we know from Equation 21 that

$$\mathrm{Pc}(\mathrm{Bnd}^1) = \frac{\sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^1)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^1))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}^0))}. \tag{26}$$

Now the only states possible for a Bnd are either $\mathrm{Bnd}(0)$ or $\mathrm{Bnd}(1)$. Thus

$$\sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^0)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^0)) + \sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^1)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^1)) = \mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd})) \tag{27}$$

or,

$$\sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^0)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^0)) = \mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd})) - \sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^1)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^1)). \tag{28}$$

Then

$$\mathrm{Pc}(\mathrm{Bnd}^0) = P_{\mathrm{Bnd}^0} = \frac{\sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^0)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^0))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}))} \tag{29}$$

$$= \frac{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd})) - \sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^1)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^1))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}))} \tag{30}$$

$$= 1 - \frac{\sum_{\forall\,\mathrm{Cl}(\mathrm{Bnd}^1)\in\mathrm{Tcl}} \mathrm{Size}(\mathrm{Cl}(\mathrm{Bnd}^1))}{\mathrm{Size}(\mathrm{Nwk}(\mathrm{Bnd}))}. \tag{31}$$

QED.

Theorem 4 $\forall\,\mathrm{Nwk}(\mathrm{Bnd})$ $\mathrm{Frf}(\mathrm{Nwk}(\mathrm{Bnd}))$ *is a free-flowing network,* $\mathrm{Cgd}(\mathrm{Nwk}(\mathrm{Bnd}))$ *is a congested network and* $\mathrm{Stp}(\mathrm{Nwk}(\mathrm{Bnd}))$ *is a stand-still.*

Proof
Consider a traffical network with $\mathrm{Pc} < 0.5$ first as $\mathrm{Pc}(\mathrm{Bnd}^1)$ in $\mathrm{Nwk}(\mathrm{Bnd}^0)$ and then, as $\mathrm{Pc}(\mathrm{Bnd}^0)$ in $\mathrm{Nwk}(\mathrm{Bnd}^1)$. This is shown in Figure 8.

From Theorem 2 and Theorem 25 we have $a = d$ and $b = c$ in Figure 8. Then from Definition 18,

$$\mathrm{Frf}(\mathrm{Nwk}(\mathrm{Bnd})) = \{\mathrm{Nwk}(\mathrm{Bnd}) : p \in [0, a)\}, \tag{32}$$

$$\mathrm{Cgd}(\mathrm{Nwk}(\mathrm{Bnd})) = \{\mathrm{Nwk}(\mathrm{Bnd}) : p \in [a, 1-a]\} \tag{33}$$

and

$$\mathrm{Stp}(\mathrm{Nwk}(\mathrm{Bnd})) = \{\mathrm{Nwk}(\mathrm{Bnd}) : p \in (1-a, 1]\}. \tag{34}$$

Then from Figure 8 $\mathrm{Frf}(\mathrm{Nwk}(\mathrm{Bnd}))$ represents the situation where cars has not yet percolated while space has. From Definition 14 this represents a free-flowing network. Similarly $\mathrm{Cgd}(\mathrm{Nwk}(\mathrm{Bnd}))$ represents that situation where both cars and space have percolated and, $\mathrm{Stp}(\mathrm{Nwk}(\mathrm{Bnd}))$ that

situation where only cars have percolated. From Definition 14, Cgd(Nwk(Bnd)) is a congested network and Stp(Nwk(Bnd)) a stand-still. Now proceed on in the same line as above for the case where Pc = 0.5 and that where Pc > 0.5 which are shown diagramatically respectively as Figure 9 and Figure 10.

In the case represented by Figure 9 Definition 18 gives

$$\mathrm{Frf}(\,\mathrm{Nwk}(\,\mathrm{Bnd})) = \{\,\mathrm{Nwk}(\,\mathrm{Bnd}) : p \in [0, 0.5)\}\,, \tag{35}$$

$$\mathrm{Cgd}(\,\mathrm{Nwk}(\,\mathrm{Bnd})) = \{\,\mathrm{Nwk}(\,\mathrm{Bnd}) : p \in [0.5, 0.5] \quad \mathrm{or\,Pc} = 0.5\} \tag{36}$$

and

$$\mathrm{Stp}(\,\mathrm{Nwk}(\,\mathrm{Bnd})) = \{\,\mathrm{Nwk}(\,\mathrm{Bnd}) : p \in (0.5, 1]\}\,. \tag{37}$$

And Definition 14 says that Frf(Nwk(Bnd)) is a free-flowing network, Cgd(Nwk(Bnd)) is a congested network and, Stp(Nwk(Bnd)) is a stand-still. The juxtaposition between the congestion of space and cars is shown in Table 11. And in the last case of Figure 10, Definition 18 again gives Equations 32, 33, and 34 while, Definition 14 again says that Frf(Nwk(Bnd)) is a free-flowing network, Cgd(Nwk(Bnd)) is a congested network and, Stp(Nwk(Bnd)) is a stand-still. The congestion of space and cars can then be shown in Table 12. QED.

Algorithm

The algorithm for finding the critical probability for a bond network is shown in the diagram in Figure 13.

Examples

This section shows the results of simulation on traffic networks. Both Figure 14 and Figure 15 are the traffic networks of Bangkok but larger area is covered by Figure 14.

The critical probability obtained from simulations similar to these could be used to determine the states of traffic congestion of that city. Once the state of the traffic at any moment is known it will be possible control the traffic accordingly. For example, once the number of cars in any area has nearly reached the percolation threshold it would be wise to stop traffic going into that area until the density has somewhat been reduced. Another example is that when there are two or more neighbouring areas experiencing heavy congestion it would be good to manage the traffic in such a way that moves cars away from those streets between those area as well as away from those congested area. This is to prevent the congested areas from merging which could cause percolation.

conclusion

Like the formation of galaxies when primordial gas has density exceeding certain value, and many things analogous to this which are found in plenty in nature, traffic in any city will come to a stand-still once the number of conjested roads reaches a percolating value which is specific for each city.

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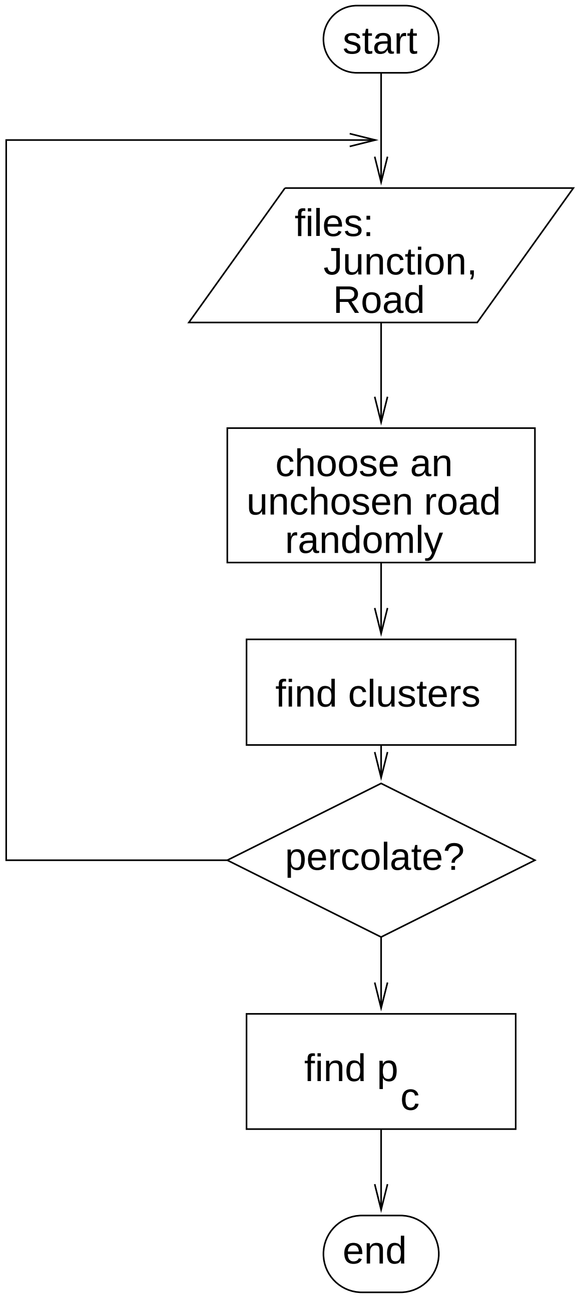

Figure 1: *Procedure for finding the critical parameter of a traffic networks.*

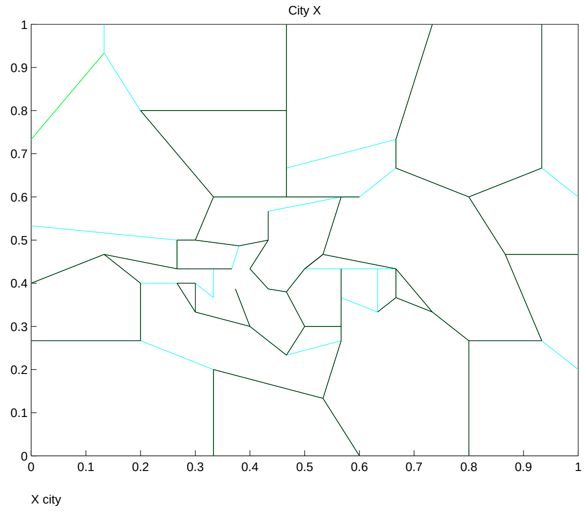

Figure 2: *Diagram representing a traffical networks of City X.*

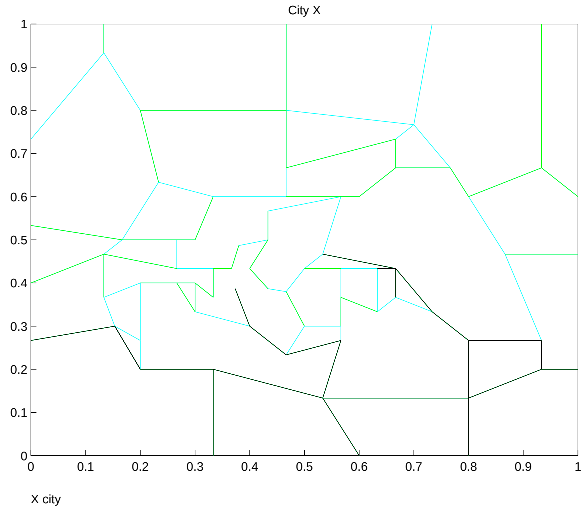

Figure 3: *Diagram representing a traffical networks of City X after the addition of a ring-road.*

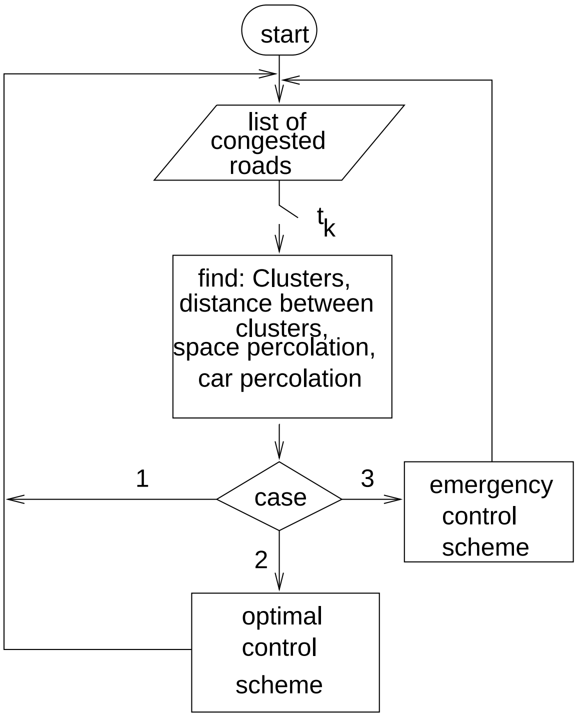

Figure 4: *Traffic control scheme for City X.*

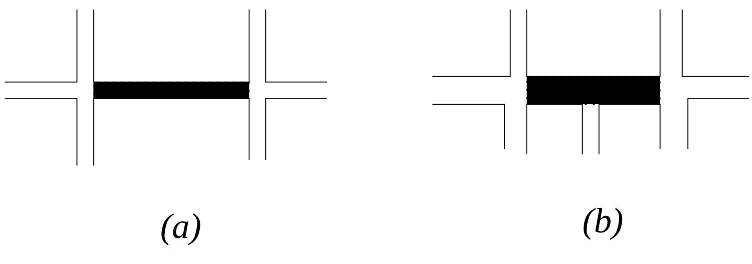

Figure 5: *The dark line shown in (a) is a road while that shown in (b) is not.*

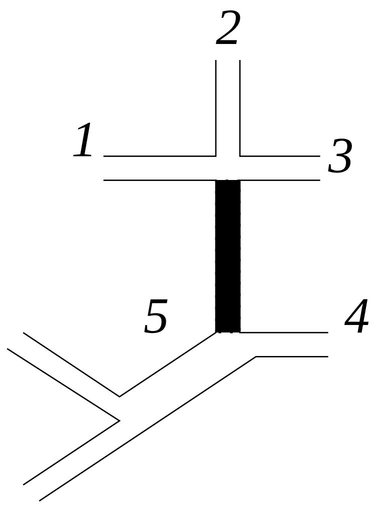

Figure 6: *The road portrayed is congested whenever no cars could enter it from any of the five adjacent roads.*

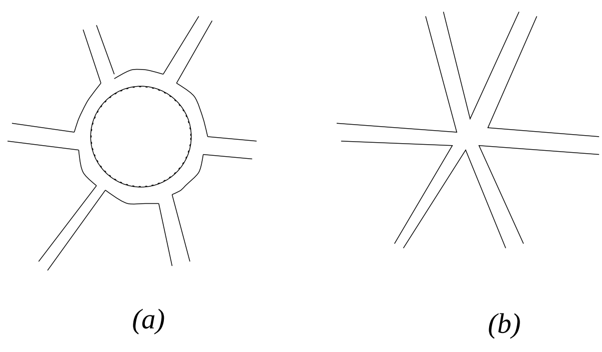

Figure 7: *(a) is considered to be the same as (b).*

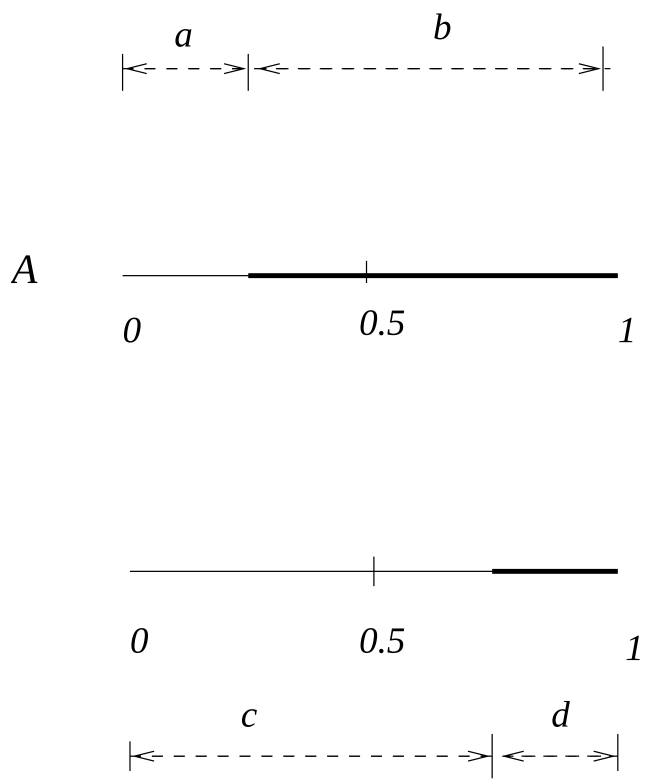

Figure 8: *The case where* Pc $<$ 0.5. *Here a is the situation where cars have not yet percolated; b is that where cars have percolated; c is that where space has percolated and d is that where space has not yet percolated.*

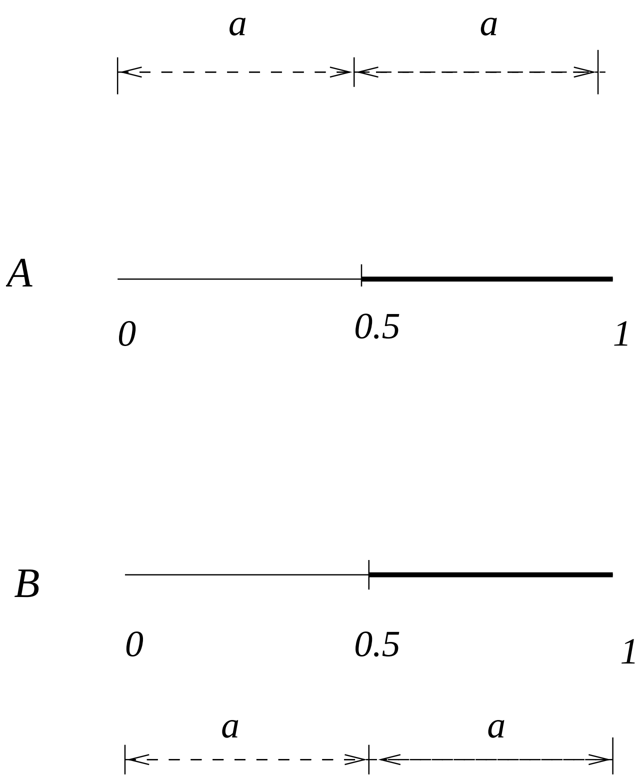

Figure 9: *The case where* Pc = 0.5.

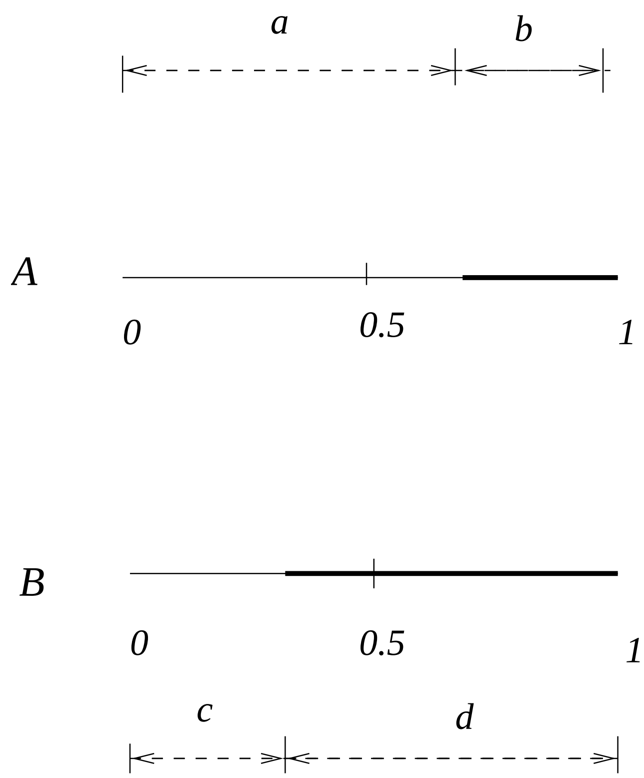

Figure 10: *The case where* Pc > 0.5.

Intervals	Cars percolation	Space percolation
Frf(·)	N	Y
Cgd(·)	Y	Y
Stp(·)	Y	N

Figure 11: *The case where* Pc = 0.5.

Intervals	Cars percolation	Space percolation
Frf(·)	N	Y
Cgd(·)	N	N
Stp(·)	Y	N

Figure 12: *The case where* Pc > 0.5.

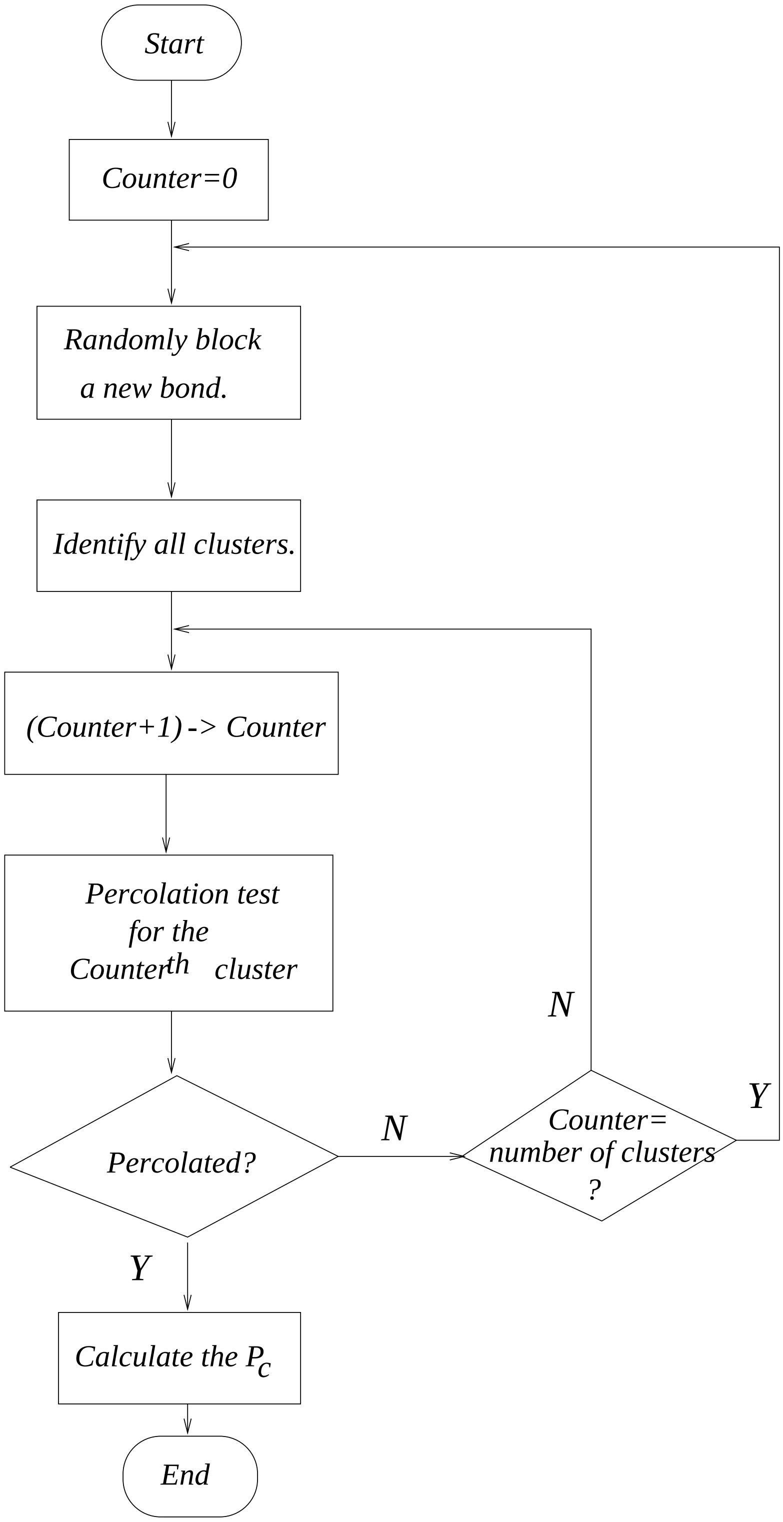

Figure 13: *Algorithm for finding a bond percolation.*

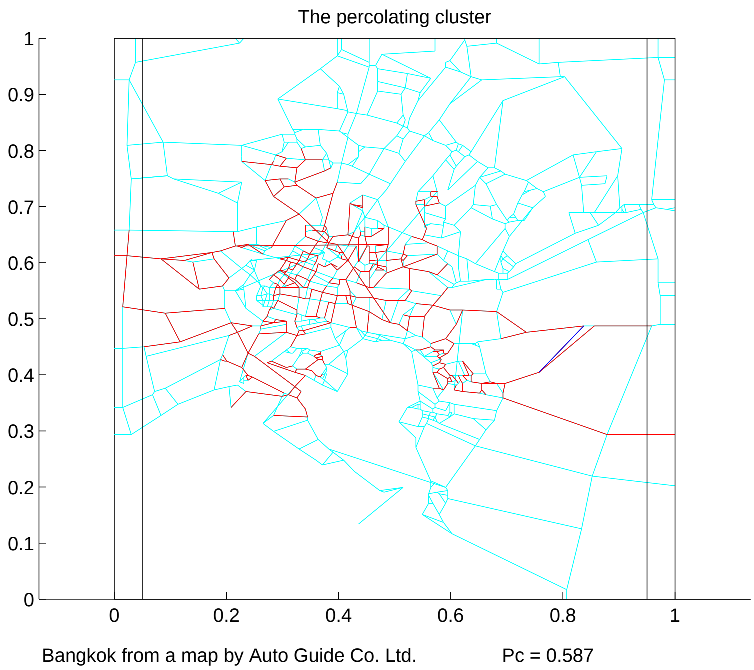

Figure 14: *Bangkok's traffic percolation simulation.*

Figure 15: *Bangkok's inner city traffic percolation simulation.*